Grade 8
TN Lesson:
Graphing Linear Inequalities in Two Variables

Britannica® Mathematics in Context

Use with MiC unit *Graphing Equations*
after page 35
OR
Use with MiC unit *Algebra Rules!*
after page 32

TN Standard: MA.8.SPI 0806.3.3
Solve and graph linear inequalities in two variables.

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Graphing Linear Inequalities in Two Variables

Support for the City Center group is growing. For this reason, the developers have decided to propose only plans that have at least 25 total units.

The Parkway group is not sure why the developers made this decision, so they choose to investigate it themselves. They begin by writing the constraint in a more mathematical way:

Houses + Town Houses ≥ 25

1. Find five plans that satisfy this constraint.

2. a. Draw and label the dividing line for this constraint on **Student Activity Sheet 1** or on graph paper.
   b. Shade the region that contains all possible plans with at least 25 units.

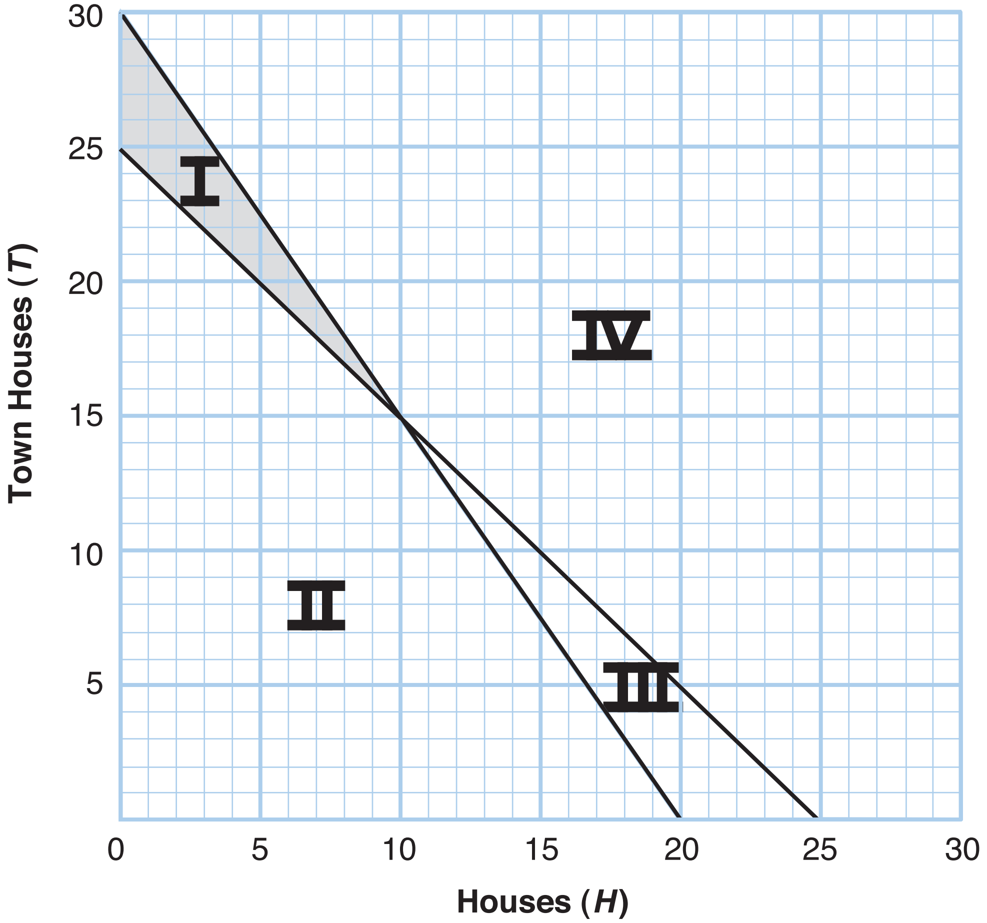
Remember, there is still the area constraint to deal with. Any feasible plans need to satisfy both the area constraint and the constraint about the total number of units.

3. a. Draw and label the dividing line for the area constraint (12,000 square meters total area) on your graph from problem **2**.
   b. Shade the feasible region for this constraint.

4. a. Find some plans that satisfy both the number constraint and the area constraint.
   b. Outline the region that is feasible for both constraints.

5. Why do you think the developers set a minimum for the total number of units, rather than setting a minimum for town houses?

Here is the complete diagram for area and number constraints.



This diagram has been copied for you on **Student Activity Sheet 2**.

6. There are two lines on the diagram dividing it into four regions (I, II, III, IV). What do the four regions mean in terms of the plans?

7. **a.** How many feasible plans are there that satisfy both constraints?

   b. Which plan would the Parkway group like best?

   c. The developers propose a plan of $H = 5$ and $T = 21$, but the City Center group demands more units. Can the developers meet that demand to some degree?

The developers reconsider their decision. They wonder what would happen if they proposed a total of 23 houses and town houses instead of the original 25.

8. Discuss the consequences.

City Regulation

In the middle of the debate, a lawyer for the city suddenly reveals that there is a city regulation that no more than half of all new units can be town houses.

9. a. Find three plans that satisfy this constraint.

b. On your graph showing the constraints for area and number of units, draw the dividing line for plans that satisfy the city regulation. Why will the regulation cause a problem?

Now the developers realize that one of the constraints must be thrown out or revised.

Here are the three constraints again:

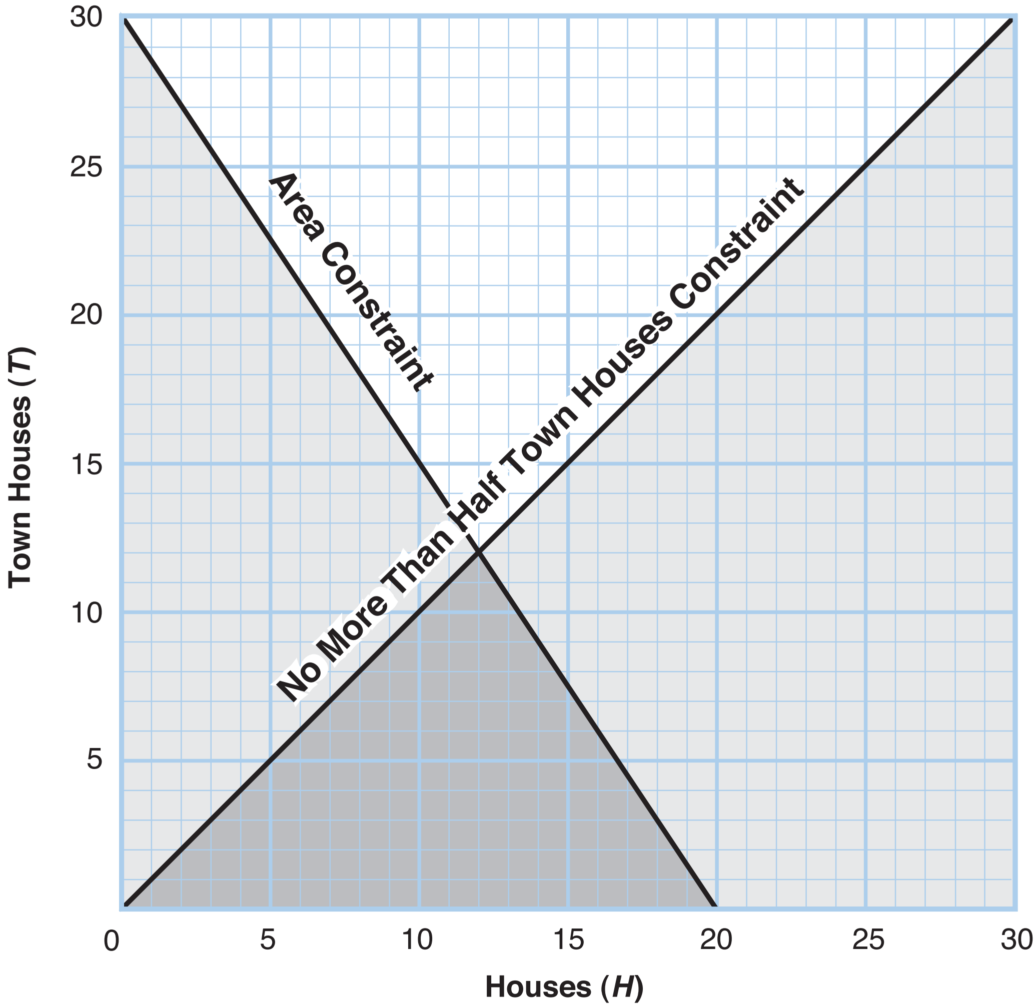
Constraint i: The available area is 12,000 square meters.

Constraint ii: The total number of units is at least 25.

Constraint iii: The number of town houses is no more than half of the total number of units.

10. How would you rank the constraints in order of importance?

Photo: Jaume Gaul—age fotostock/Imagestate



11. a. How can Constraint ii be changed in order to make one or more plans possible? What would a good plan be? Use **Student Activity Sheet 3** for your work.

b. What is the best plan for Parkway now?

c. for City Center?

More Land

Demolition of a warehouse has made it possile to enlarge the available land by an additional 1,400 square meters. Now, new plans are feasible.

The developers decide to compare each new plan to the plan (12, 12).

12. **a.** Why are the developers starting with the plan (12, 12)?

b. Which plans are feasible now?

13. Which of the new plans is best for the Parkway group? for the City Center group?

A Graphing Linear Inequalities in Two Variables

Summary

In real situations, there is usually more than one constraint. Each constraint splits a diagram into two regions. The region that is feasible for the first constraint can be split by the second constraint, and so on.

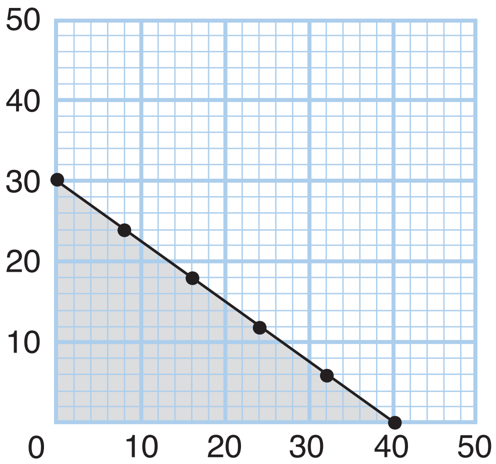
In this lesson, the feasible region became smaller and smaller. This means there are fewer possible plans. Sometimes, however, constraints can be changed to allow for more possibilities.

Check Your Work

The developers settled on (13, 13), a good solution for both neighborhoods, and a solution that was best for the city as a whole, too.

14. Why is (13, 13) a good solution?

15. On the right is a graph showing the dividing line for a constraint. Copy this graph and label as many things as you can to show what you have learned in this unit.



16. On **Student Activity Sheet 1** or on graph paper, draw a picture of a feasible region with three constraints. Draw the dividing lines of two other constraints that have no effect on the feasible region.

Name ______________________________

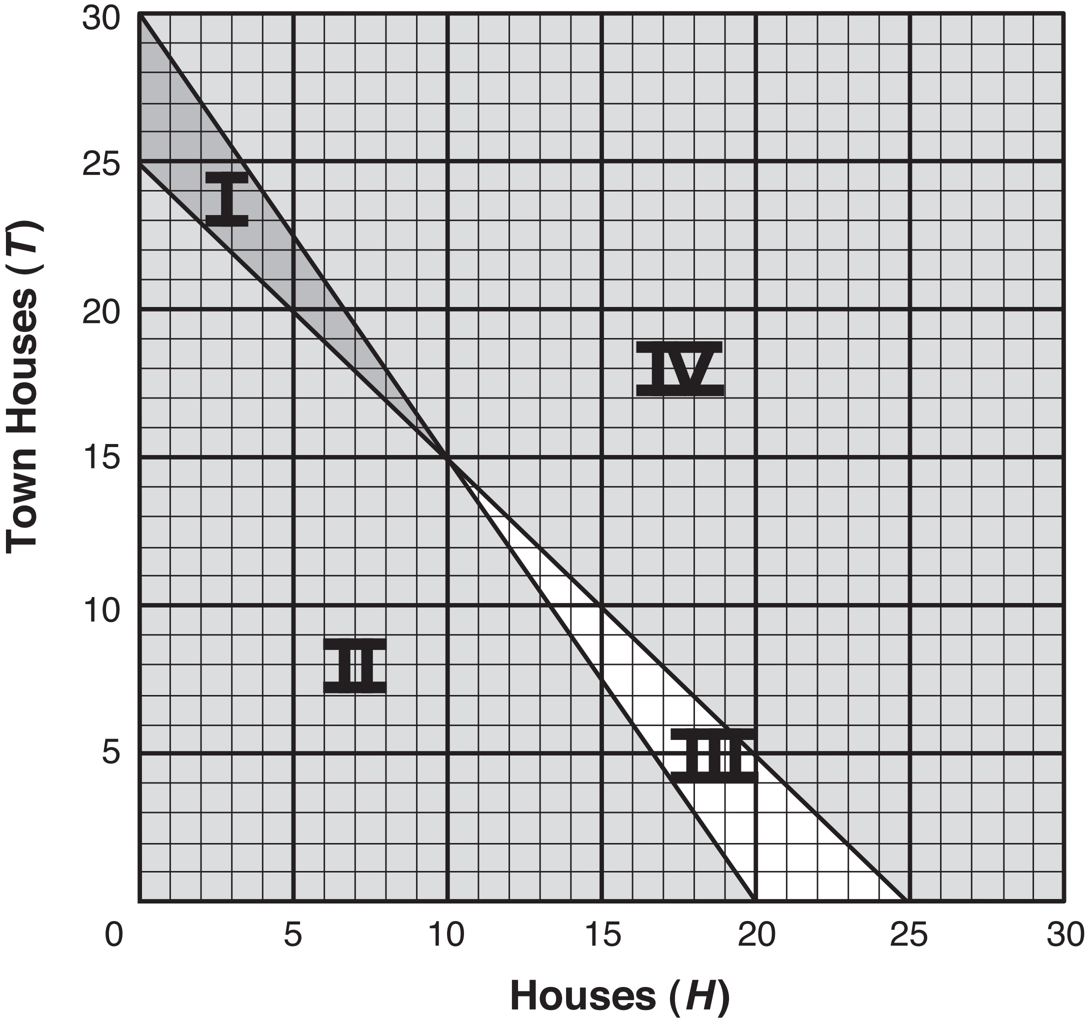
Student Activity Sheet 1

Use with *Graphing Linear Inequalities in Two Variables*, pages 1 and 6.

Student Activity Sheet 2

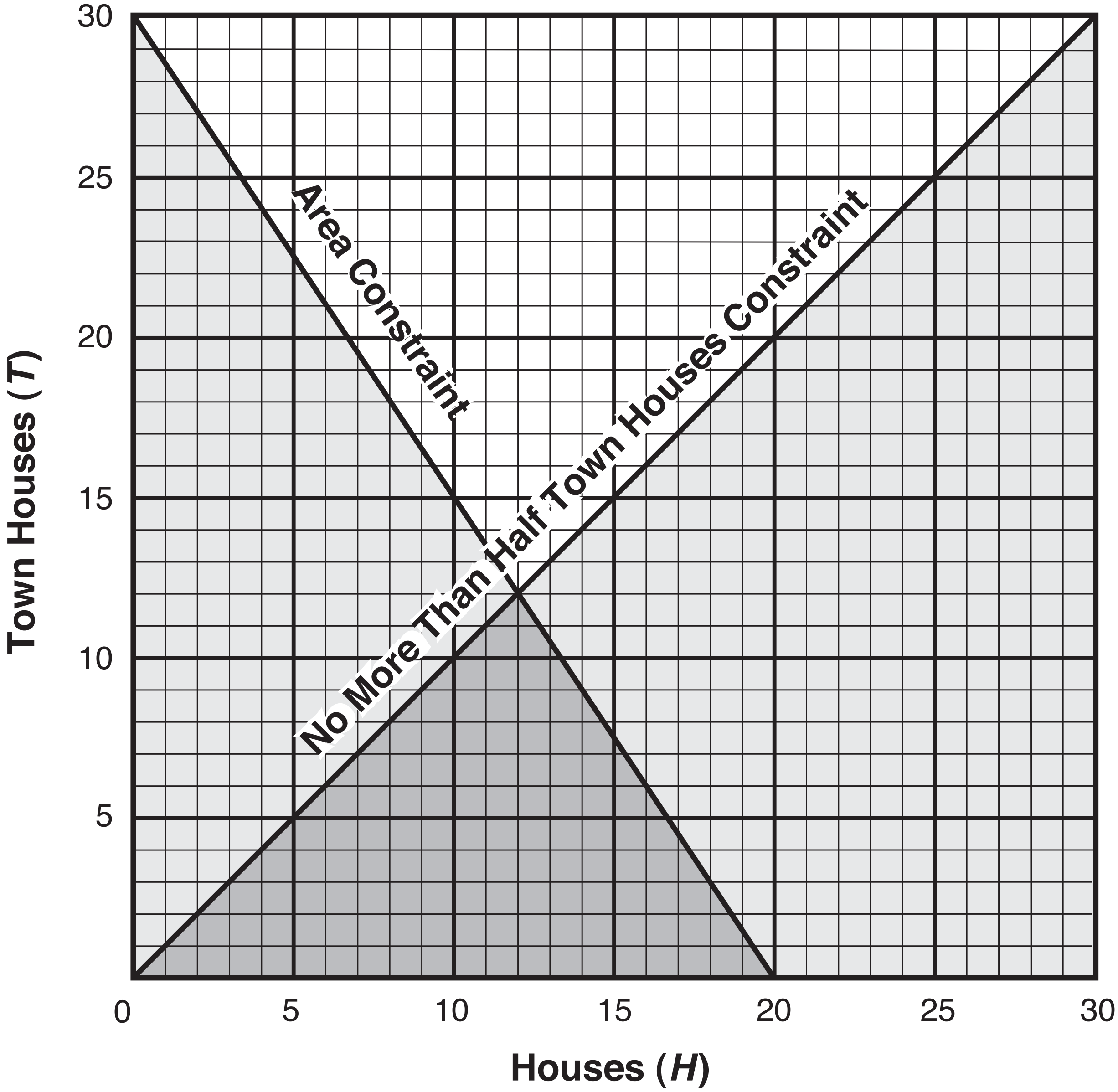
Use with *Graphing Linear Inequalities in Two Variables*, pages 2 and 3.

Name ______________________________



Name ____________________________________

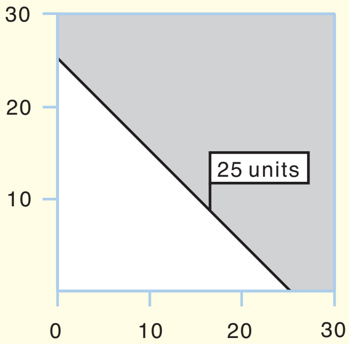
Use with *Graphing Linear Inequalities in Two Variables*, page 4.



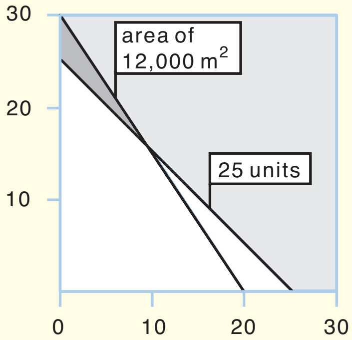
Solutions and Samples

1. Answers will vary. Sample responses: (7, 25), (4, 24), or (101, 23), (24, 24), or (0, 25). Accept any response that shows a total of 25 or more housing units.

2. **a–b.**

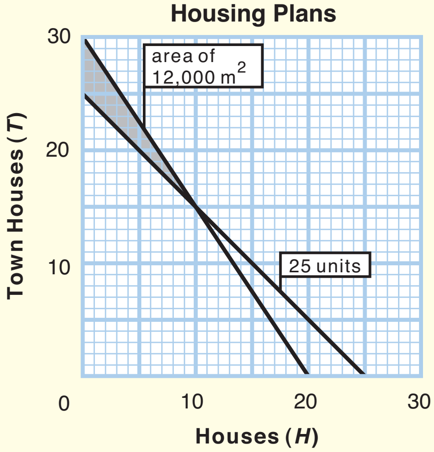


3.



4. **a.** Any point that is *below* the 12,000 m^2 line and *above* the 25 units line is acceptable. Two such points are (5, 22) and (10, 15).

   b.



5. Answers will vary. Sample response:

   Asking for more town house units would have made the Parkway group angry. By wording their decision the way they did, the developers actually get more town houses without alarming the Parkway group.

Hints and Comments

Materials

Student Activity Sheet 1 (one per student); graph paper, optional (one sheet per student)

Overview

Students draw two different constraints on one graph and take both constraints into account to determine feasible plans that meet both conditions.

About the Mathematics

The problem here involves two different constraints. Each new constraint requires an additional dividing line when graphing the situation. Constructing graphs in an organized and neat way allows students to graph the constraint's dividing lines accurately to easily see the feasible region that meets the conditions of the constraints.

Students may want to abbreviate *houses* and *town houses* by writing shortcuts as "H" and "T". That is fine as long as students realize that the letters stand for the *number* of houses and town houses.

Planning

Students may work on problems **1–5** in small groups or pairs. Have a discussion of problems **1–5** using the graph shown on page 2. Problem **4** may be used as informal assessment.

Comments About the Solutions

1. In this problem, students need only focus on the one given constraint introduced here.

2.–3. A constraint can be drawn as a dividing line. Students should understand this by now.

4. **Informal Assessment**

   This problem assesses students' ability to plot points and draw graphs (straight lines); to understand and know how to find feasible and unfeasible regions and to find the value of a given combination of items and find a combination for a given value.

5. You might ask students to go back to the original context of this problem to see if their solutions and explanations make sense within the context. The ability to translate mathematics back to the terms of the original story problems is an important goal in this lesson.

Solutions and Samples

6. Region I is the feasible region, where the plans meet both constraints; the area is 12,000 m^2 or less and the total number of housing units is 25 or more.

Region II is where the plans meet the area constraint, but not the total number of houses constraint.

Region III is where the plans meet neither constraint.

Region IV is where the plans meet only the total number of houses constraint.

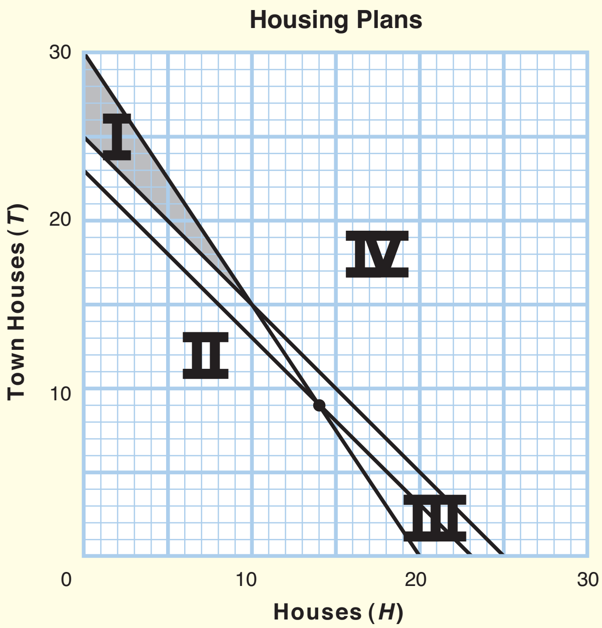
7. a. There are 36 feasible plans that satisfy both constraints. Points on the line represent feasible plans as well as points inside Region I.

b. Answers will vary. Some students may see that the plan (10, 15) has the most house units, so it will probably be the most popular with the Parkway group.

c. Yes. The plan (5, 22) is within the feasible region and allows one more town house unit.

8. Answers will vary. Changing the total number of housing units to 23 changes the number constraint. The dividing line for this constraint must move down and to the left. The new line passes through (0, 23) and (23, 0). This allows many more plans to be within the feasible region. The new "crossing point" is at (14, 9).

The feasible region (Region I) becomes much larger.



Hints and Comments

Materials

transparency of Student Activity Sheet 2, optional (one per class); overhead projector, optional (one per class)

Overview

Students continue investigating problems that involve two constraints and relate the mathematical representation to the problem situation. They consider the impact of a change in the constraint on the total number of units.

About the Mathematics

The graph shown on page 2 can be made more meaningful by adding labels to identify each constraint's dividing lines and adding a key that could be used to identify the four regions. Each of the four regions can be described using words (See the solutions column for problem **6**) and by using a mathematical inequality:

- Region I: $600\,H + 400\,T \le 12{,}000$ m^2 and $H + T \ge 25$
- Region II: $600\,H + 400\,T \le 12{,}000$ m^2 and $H + T \le 25$
- Region III: $600\,H + 400\,T \ge 12{,}000$ m^2 and $H + T \le 25$
- Region IV: $600\,H + 400\,T \ge 12{,}000$ m^2 and $H + T \ge 25$

In these inequalities, the H and T stand for the number of houses and town houses. Most students will be familiar with this notation, having worked with it in the unit *Comparing Quantities* and in other algebra strand units.

Planning

You may want to make a transparency of Student Activity Sheet 2 and use it on the overhead projector to discuss students' solutions for problems **1–8**. Students may work in pairs or in small groups on problems **6–8**.

Comments About the Solutions

6. Students need to count all the plans in the region, including those on the dividing line.

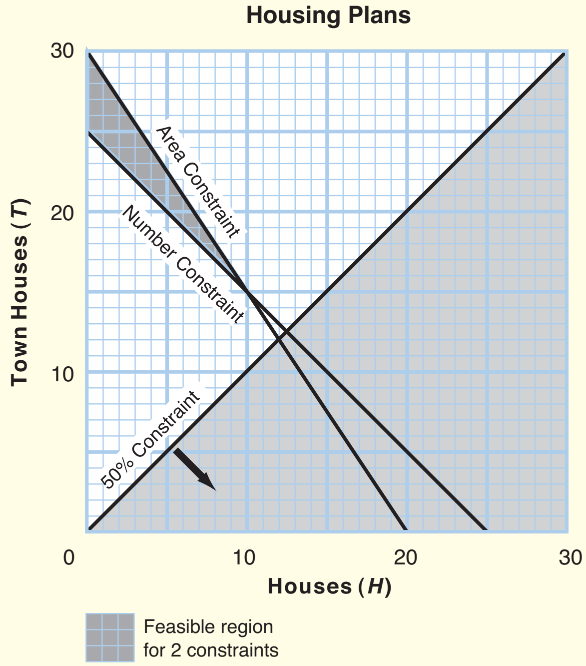
8. The consequences can be discussed in terms of the context as well as the graph. As students have seen, the dividing line shifts when a constraint changes. You can visualize this by using a straight line on the same transparency described above to show how the dividing line that represents housing units shifts.

Solutions and Samples

9. a. Answers will vary. Accept any plan that satisfies the constraint and has the same number (or less than the same number) of town houses as houses. Sample response:

Three typical plans are (10, 10), (20, 20), and (30, 30). Note that these plans satisfy only this constraint and lie on the dividing line.

b.



Explanations will vary. Sample explanation:

To satisfy the city regulation, all plans should be on or below the new dividing line. The regulations will cause a problem because no plan can satisfy all three constraints. Region I is above the new dividing line.

10. Answers will vary. Sample response:

Constraint iii is the most important, as it is a law. Constraint i (the area constraint) is also important, but could be changed if the city would give more land to the two groups. Constraint ii (total number of units) is the most flexible.

Hints and Comments

Materials

Student Activity Sheet 2 (one per student)

Overview

Students learn about a new constraint for the housing plans and find three plans that satisfy the conditions of the new constraint. They then rank the three given constraints in order of importance.

About the Mathematics

The new constraint introduces a dividing line that lies or slants in the opposite direction as the first two dividing lines: it slants from the lower bottom left-hand corner and extends to the upper right-hand corner of the graph.

In general, as more constraints are added, it becomes increasingly more difficult to identify a plan that meets the conditions of each constraint. The problem here models those often found in real-life situations. Most students will be able to solve such problems after analyzing and interpreting the graph that illustrates the given constraints. In reality, the outcome of a debate, such as that depicted in this unit, is often a compromise plan that tries to meet most of the constraints and satisfy most of the demands of the parties involved.

Planning

Students may work on problems **9** and **10** in pairs or in small groups. These problems may also be assigned as homework.

Comments About the Solutions

9. a. Some students may need help in understanding the new constraint. You might mention to students that since the constraint mandates that no more than $\frac{1}{2}$ of all the housing units can be town houses, if the number of town houses makes up 50% of the total units, there must be an equal number of town houses and houses in the plan.

10. From a mathematical standpoint, there is no way to rank the three constraints in order of importance; each is equally important. Because of the given context here, this is no one correct answer.

Solutions and Samples

11. a. Answers will vary. Sample responses:

- Move the dividing line for Constraint ii down and to the left to make more feasible plans.
- Decrease the total number of housing units in Constraint ii down to 24 units or lower.
- Change the housing plan to (12, 12) to make a total of 24 housing units.

b. The best plan for Parkway is still one that calls for the greatest number of family houses that still satisfies all three constraints. Specific answers will depend on the changes students made to Constraint ii in part **a** above.

c. The best plan for City Center is still one that calls for the greatest number of town houses that still satisfies all three constraints. Specific answers will depend on the changes students made to Constraint ii in part **a** above.

Hints and Comments

Materials

Student Activity Sheet 3 (one per student); transparency of Student Activity Sheet 3, optional (one per class); overhead projector, optional (one per class)

Overview

Students continue to investigate the impact of the new constraint on the original two constraints.

Planning

Students may work on problem **11** in pairs or in small groups. This problem and the Extension activity below may also be assigned as homework. You may want to make a transparency of Student Activity Sheet 3 to discuss students' solutions for problem **11**.

Comments About the Solutions

11. Homework

This problem may be assigned as homework. You may need to remind some students that the "best" plan for members of the Parkway group would include as many family houses as possible and the "best" plan for members of the City Center would include as many town houses as possible.

Extension

You might challenge students to find the highest possible percent of total units that could be town houses and still satisfy the conditions of the other constraints to result in a feasible plan. [In this context, 60% is the maximum percentage of town houses that will still meet the other constraints and result in a feasible plan.]

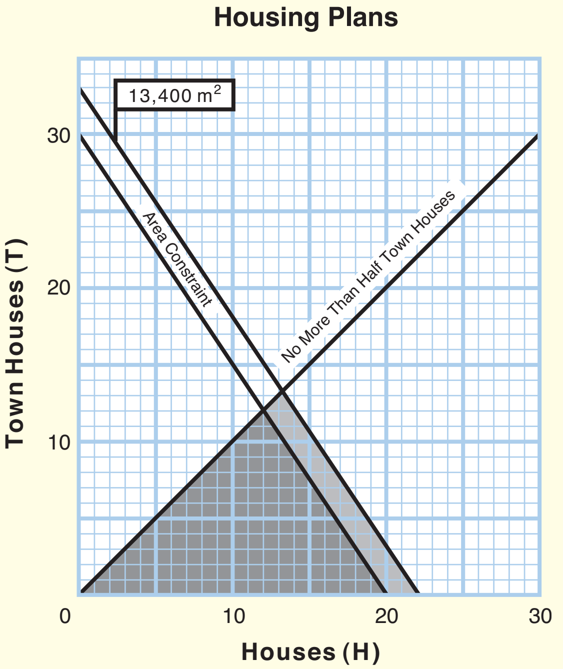
You may have interested students create three new constraints to replace the existing ones and try to find feasible plans that meet their constraints. This is a powerful activity that will strengthen students' understanding of the underlying mathematical concepts here.

Solutions and Samples

12. a. Answers will vary. Sample response:

The plan (12, 12) is a good starting point because each group gets the same number of housing units.

b. Answers will vary. The feasible plans now include the following: (12, 12), (13, 12), (14, 12), and (13, 13). Any plans that fall on or under the 50% constraint and on or to the left of the area constraint line are feasible.



13. The best plan for Parkway would be (13, 12) since that provides the greatest number of houses and the fewest number of total housing units. The best plan for City Center would be (13, 13) or (14, 12) since both plans would allow for 26 housing units to be built.

Hints and Comments

Materials

Students' completed copies of Student Activity Sheet 3 (one per student)

Overview

The constraint for the total area of available land is adjusted. Students investigate to find out how this change impacts the housing plans situation.

About the Mathematics

The mathematics explored on this page implies that a relatively minor change in one constraint can have a major impact in the overall situation. One can look at the constraint change from a strictly mathematical point of view. Drawing the new dividing line for the total area on the same graph can be used to find a new feasible region that satisfies all three constraints. However, the mathematical solution may not always be the best solution in light of the context of the situation. Students should implicitly learn that the best solution from a logical or mathematical standpoint may not be the best solution for all parties involved in a real-life problem situation.

Planning

Students may work on problems **12** and **13** individually. Problem **12** may be used as informal assessment.

Comments About the Solutions

12. Informal Assessment

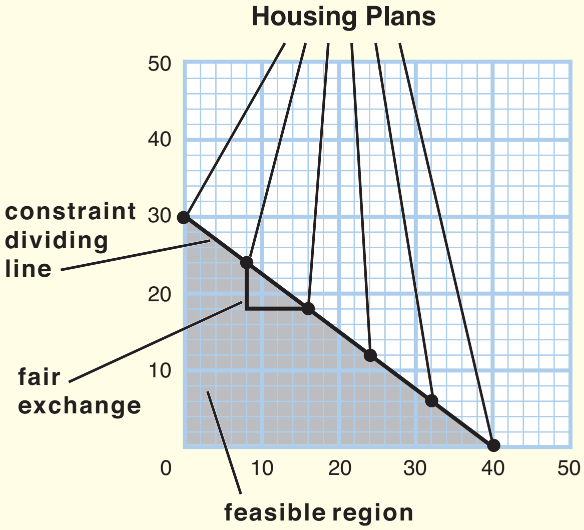
This problem assesses students' ability to understand the concept of constraint (in terms of the context and in terms of the graph); to identify the constraints in a given problem situation; to combine different kinds of information (multiple constraints) in one graph and make decisions based on the information; and to recognize that mathematics can be used to describe and clarify complex situations.

Some students might find it helpful to draw a dividing line for the new total area constraint at 13,400 m² (12,000 + 1,400). Although this is not necessary, the graph (see solutions column on the left) allows students to explore and review all aspects of the situation. There is no mathematical reason to start with the plan (12, 12).

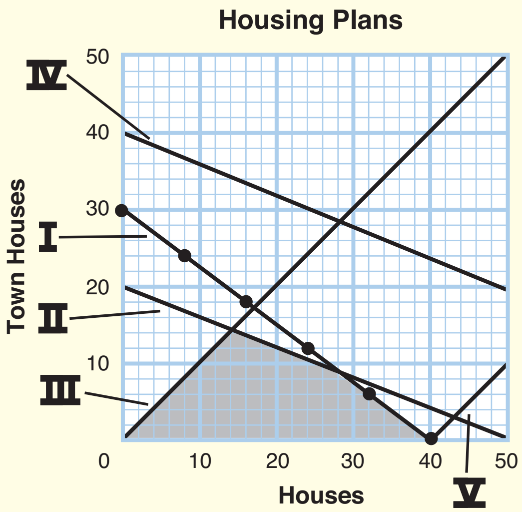
Solutions and Samples

14. Answers will vary. Some students may say that the plan (13, 13) is a good solution because it satisfies all the constraints. Others may say that this plan is good because it will make both groups fairly happy within those constraints.

15.



16. Graphs will vary. Sample graph:



In the above example, Constraints iv and v have no effect on the feasible region.

Hints and Comments

Materials

Student Activity Sheet 1, optional (two copies per student); graph paper, optional (two sheets per student)

Overview

Students read and discuss the main concepts reviewed in the Summary and solve additional problems involving more than two constraints and finding feasible plans.

Planning

Students may work on problems **14–16** individually. Problem **15** may be used as informal assessment. Problem **16** and the Extension activity below may be assigned as homework.

Comments About the Solutions

15. Informal Assessment

This problem assesses students' ability to understand the concept of fair exchange (in terms of the context and in terms of the graph); to understand the concept of constraint (in terms of the context and in terms of the graph); to understand that all of the solutions to a linear equation lie on a line; to use fair exchange as a way to draw graphs and to find new combinations; and to understand and use relationship among constraints, dividing lines, and feasible and unfeasible regions.

16. Homework

This problem may be assigned as homework. You might also have students draw a set of constraints that contradict one another.

Extension

As an extension to problem **16**, challenge students to write a story or new context that could be used to describe the graph they made. Encourage students to find new contexts, other than vacant land development, in which to base their stories. This Extension may also be assigned as homework.